Wannier-Stark ladder in the linear absorption of a random system with scale-free disorder

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We study numerically the linear optical response of a quasiparticle moving on a one-dimensional disordered lattice in the presence of a linear bias. The random site potential is assumed to be long-range-correlated with a power-law spectral density $S(k) \sim 1/k^{\alpha}$, $\alpha > 0$. This type of correlations results in a phase of extended states at the band center, provided α is larger than a critical value α_c [F. A. B. F. de Moura and M. L. Lyra, Phys. Rev. Lett. **81**, 3735 (1998)]. The width of the delocalized phase can be tested by applying an external electric field: Bloch-like oscillations of a quasiparticle wave packet are governed by the two mobility edges, playing now the role of band edges [F. Domínguez-Adame *et al.*, Phys. Rev. Lett. **91**, 197402 (2003)]. We demonstrate that the frequency-domain counterpart of these oscillations, the so-called Wannier-Stark ladder, also arises in this system. When the phase of extended states emerges in the system, this ladder turns out to be a comb of doublets, for some range of disorder strength and bias. Linear optical absorption provides a tool to detect this level structure.

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I. INTRODUCTION

Since 1998,[1] there exists an increasing interest in studying the localization properties of the quasiparticle wave functions in one-dimensional (1D) disordered systems with a long-range-correlated site potential landscape.[2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,21,22] Random sequences, having a power-law spectral density $S(k) \sim 1/k^{\alpha}$ with $\alpha > 0$, result in a phase of extended states at the band center, provided α is larger than a critical value α_c.[1,19] This finding contradicts the widely admitted conclusion of the one-parameter scaling theory of localization[23] that all states of noninteracting quasiparticles in one and two dimensions with time reversal symmetry are localized (see Refs. 24,25,26,27 for an overview). As a matter of fact, a great deal of work is being devoted to put the correlation-induced low-dimensional localization-delocalization transition (LDT) on solid grounds.

It is worthwhile to notice that long-range correlations with spectral density of the form $1/k^{\alpha}$ are widely presented in nature, both in *vitro* and in *vivo* (see, e.g., Refs. 28 and 29 and references therein). This type of correlation gives rise to the "fractal geometry of nature" introduced by Mandelbrot.[30] Importantly, in 1992 it was conjectured that the long-range power-law correlations exist in nucleotide sequences in DNA.[31,32,33,34] This conjecture opened prospectives to quantify nature *in vivo* with critical exponents.[35] It was argued also that long-range correlations in DNA sequences can explain the long-distance charge transport in these systems.[10,18,20,36] The $1/k^{\alpha}$-law has also its trace in energy level statistics.[15] All said above unambiguously testifies that studying the properties of disordered systems with long-range-correlated disorder is an attention grabbing task.

It is known since seminal papers by Bloch[37] and Zener[38] that an electron moving in a periodic potential and subjected to a uniform electric field is localized due to the Bragg reflection. It performs a periodic motion, known as Bloch oscillations,[39,40] which is characterized by an angular frequency $\omega_B = eFd/\hbar$ and a spatial extension $L_B = W/(eF)$, where $-e$ is the electron charge, F is the applied electric field strength, d denotes the spatial period of the potential, and W stands for the band width. The Bloch oscillations were observed for the first time as oscillations of electronic wave-packets in semiconductor superlattices[41,42,43,44,45,46] (see for an overview Ref. 47), and later on as a periodic motion of ensembles of ultracold atoms[48,49] and Bose-Einstein condensates[50] in tilted optical lattices. The multiplicity of observable physical phenomena related to electron Bloch oscillations increases even more when a semiconductor superlattice is subjected to joint, perpendicular or tilted, electric and magnetic fields.[51,52,53,54,55]

Recently, it was demonstrated that 1D disordered systems with the $1/k^{\alpha}$ spectral density support Bloch-like oscillations of quasiparticles.[13] It was also shown that these oscillations provide a tool to measure the energy width of the delocalized phase arising at $\alpha > \alpha_c$. More specifically, the two mobility edges, which separate the phase of extended states from the two phases of localized ones, were found to play the role of effective band edges, i.e., it is the width Δ of the mobility band who determines the spatial extension $L_B^* = \Delta/(eF)$ of Bloch oscillations. Therefore, this width can be measured in biased disordered lattices.

In this work we report further progress along this line by considering the frequency-domain counterpart of the Bloch oscillations, the so-called Wannier-Stark ladder (WSL)[56] (see Refs. 57 and 58 for brief historical surveys). The WSL is characterized by a series of equidistant quasistationary levels separated by an energy $U = \hbar\omega_B = eFd$. The progress in semiconductor growth techniques made it possible to firmly establish the existence of ladder level structures in semiconductor superlattices[59,60,61,62] as well as in δ-doped superlattices.[63,64] There exists evidence that moderate uncorrelated disorder does not destroy the required phase coherence to see the WSL in continuous (many band) models.[65] We focus on the one-band approximation and demonstrate that in our system the WSL also arises, in spite of the underlying randomness. Strong correlations in the disorder facilitate the observation of the WSL. At $\alpha > \alpha_c$, when the phase of extended states emerges at the center of the band, the WSL may present a comb of doublets, reflecting the doublet energy structure of the unbiased system.[66] To work out the problem, we numerically calculate the absorption spectrum varying the correlation exponent α, the disorder strength, and the magnitude U of the bias.

The outline of the paper is as follows. In the next section, we present our model which is based on a tight-binding Hamiltonian of a quasiparticle, moving in a long-range-correlated potential landscape and subjected to a linear bias. In Sec. III we recall the basic physics of the linear absorption spectrum in the absence of both bias and correlations in disorder. The central parts of the paper are Sec. IV and Sec. V, where the results of numerical simulations of the absorption spectrum profile in disorder-correlated systems are shown. We begin with a brief discussion of the absorption spectrum behavior upon increasing the disorder correlations in bias-free systems (Sec. IV), proceeding in Sec. V onto the absorption in biased lattices. We discuss in detail its dependence on the driving parameters of the model (bias magnitude, disorder strength and correlation exponent α) and provide an evidence of that the correlations in disorder facilitate the occurrence of the WSL. Finally, Sec. VI concludes the paper.

II. MODEL

We consider a biased tight-binding model with diagonal disorder on an otherwise regular 1D open lattice of spacing unity and N sites (N is assumed to be even). We assign to each lattice site two levels (ground and excited states) with transition energy $\mathcal{E}_n$ and consider optical transitions between them. The model Hamiltonian is

$$\mathcal{H} = \sum_{n=1}^{N} \left[\mathcal{E}_n - U\left(n - \frac{N}{2}\right) \right] |n\rangle\langle n| - \sum_{n=1}^{N-1} \Big(|n\rangle\langle n+1| + |n+1\rangle\langle n| \Big) . \qquad (1)$$

Here, $|n\rangle$ denotes the state in which the n-th site is excited, whereas all the other sites are in the ground state. The energy of the state $|n\rangle$, $\mathcal{E}_n = \bar{\mathcal{E}} + \varepsilon_n$, is assumed to have a stochastic part ε_n generated according to[1]

$$\varepsilon_n = \sigma C_\alpha \sum_{k=1}^{N/2} \frac{1}{k^{\alpha/2}} \cos\left(\frac{2\pi k n}{N} + \phi_k \right) , \qquad (2)$$

where $C_\alpha = \sqrt{2}\big(\sum_{k=1}^{N/2} k^{-\alpha}\big)^{-1/2}$ is the normalization constant and $\phi_1, \ldots, \phi_{N/2}$ are $N/2$ uncorrelated random phases uniformly distributed within the interval $[0, 2\pi]$. The distribution (2) has zero mean $\langle \varepsilon_n \rangle = 0$ and standard deviation $\langle \varepsilon_n^2 \rangle^{1/2} = \sigma$, where $\langle \ldots \rangle$ indicates averaging over realizations of random phases ϕ_k. The quantity σ will be referred to as magnitude of disorder. The stochastic sequence (2) is characterized by a correlation function

$$\langle \varepsilon_n \varepsilon_m \rangle = \frac{\sigma^2 C_\alpha^2}{2} \sum_{k=1}^{N/2} \frac{1}{k^\alpha} \cos\left[\frac{2\pi k (n-m)}{N} \right] . \qquad (3)$$

which is long-ranged, except for the particular value of the exponent $\alpha = 0$, when $\langle \varepsilon_n \varepsilon_m \rangle = \sigma^2 \delta_{mn}$, δ_{mn} being the Kronecker δ. The site energies in this case are uncorrelated. Correlations, however, arise as soon as $\alpha \neq 0$. Thus, for $\alpha < 1$, they are power-law-like, i.e., the correlation function (3) decays as $|n-m|^{\alpha-1}$. If $\alpha \gg 1$, the term with $k = 1$ in the series (3) is dominant, and $\langle \varepsilon_n \varepsilon_m \rangle = (1/2)\sigma^2 C_\alpha^2 \cos[2\pi(n-m)/N]$, implying full correlation of the on-site energies.

The term $-U(n - N/2)$ in the Hamiltonian (1) describes the linear bias. We will not necessarily relate it to the presence of an external uniform electric field, as in the case of an electron moving in the conduction band.[67] The bias can also be an intrinsic property of the system, as it takes place in dendritic species (see, e.g., Ref. 68 and references therein). In this case, the Hamiltonian (1) models a 1D Frenkel exciton in a disordered lattice with energetic bias. Finally, the intersite transfer integrals in (1) are restricted to nearest-neighbors, and it is set to -1 over the entire lattice. Also, we set $\bar{\mathcal{E}} = 0$ hereafter without loss of generality.

As we already mentioned in the Introduction, in the absence of bias ($U = 0$), the above model supports a phase of extended states at the center of the band, provided the correlation exponent α is larger than a critical value α_c. At $\alpha < \alpha_c$ all the states are localized, which implies that the model under consideration undergoes an LDT with respect to the correlation exponent α. In Ref. 1, where the above model of disorder was introduced, the disorder magnitude was set to $\sigma = 1$, and the critical value α_c was found to be $\alpha_c = 2$. It may seem that α_c depends on σ. However, it was demonstrated later on that $\alpha_c = 2$ is the universal critical value for the LDT to occur in this model, independently of σ.[19]

Another peculiarity of this model, having a direct relationship to the specific form of the random potential (2)

as well as to its localization properties, is that the absorption spectrum at $\alpha > \alpha_c = 2$, i.e., in the presence of the phase of extended states, reveals a double-peaked structure (see Ref. 66 and Sec. IV).

III. ABSORPTION SPECTRUM

The quantity subjected to calculation throughout this paper will be the absorption spectrum defined as

$$A(E) = \frac{1}{N}\left\langle \sum_{\nu=1}^{N} \left(\sum_{n=1}^{N} \psi_{\nu n} \right)^2 \delta(E - E_\nu) \right\rangle , \qquad (4)$$

where the E_ν and $\psi_{\nu n}$ are the eigenenergies and eigenfunctions, respectively, obtained after diagonalization of the Hamiltonian (1). The quantity $F_\nu = \left(\sum_{n=1}^{N} \psi_{\nu n} \right)^2$ is the dimensionless oscillator strength of the ν-th state.

In order to gain insight into the effects of long-range correlations in disorder and the bias on the absorption, we first recall the basic features of the absorption spectrum in the absence of both ($\alpha = 0$ and $U = 0$). Uncorrelated diagonal disorder results in localization of all the states[23] and in the appearance of Lifshits tails in the density of states (DOS), outside the bare quasi-particle band which ranges from $E = -2$ to $E = 2$. Since we set negative sign of the hopping integral, the low-energy part of the DOS (with E around -2) is of importance for the linear optical absorption. The majority of states lying in this region are localized at different (weakly overlapped) segments. Some of them are bell-like, i.e., without nodes within localization region, while the other and higher (band) states resemble standing waves with nodes.[69] The bell-like states dominate the optical absorption because they accumulate oscillator strengths large compared to those of the other states. They result from localization on fluctuations of the site potential, which have a well-like envelop.[70] The typical size of a such potential wells, N^*, determines the extension of the bell-like states, while the depth $\sigma/\sqrt{N^*}$ governs the width of the absorption spectrum (see below). Notice that the potential depth is $\sqrt{N^*}$ times as small as the bare magnitude σ. This effect is known as the exchange (or motional) narrowing: fluctuations of a stochastic site potential of alternating signs are averaged out by a quasiparticle rapidly moving (due to a large exchange interaction $J \gg \sigma$) within a region of size N^*.[69,71,72,73,74]

The absorption spectrum is peaked slightly below the bare band edge $E = -2$ and represents an inhomogeneously broadened line with a Gaussian-like red and a Lorentzian-like blue tail (see, e.g., Ref. 71). The localization size of the optically dominant states, N^*, and the full width at half maximum (FWHM) of the absorption, σ^*, are estimated as[72]

$$N^* = \left(\frac{3\pi^2}{\sigma} \right)^{2/3} , \qquad (5a)$$

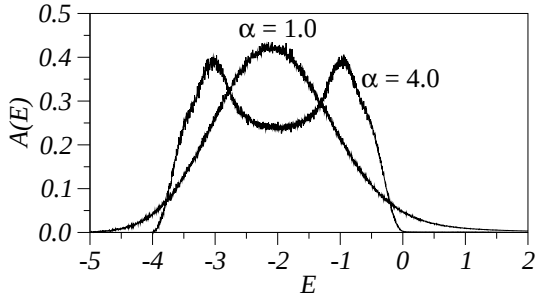

FIG. 1: Absorption spectra of an ensemble of unbiased chains ($U = 0$) with $N = 500$ sites calculated for two values of the correlation exponent α, shown in the plot. In both cases, the magnitude of disorder is $\sigma = 1$. Each curve were obtained after averaging over 3×10^4 realizations of disorder.

$$\sigma^* = \frac{2\sigma}{\sqrt{N^*}} = 6\pi^2 \left(\frac{\sigma}{3\pi^2} \right)^{4/3} . \qquad (5b)$$

To conclude this section, note that both short-range and power-law long-range correlations in disorder result in decreasing the localization length of the band edge states[22,74] and, subsequently, increasing the absorption line width.[66,74] In contrast, this is not the case for correlations which are stronger than power-law-like. An example is a sequence (2) at $\alpha \gg 1$ (see the next section).

IV. UNBIASED SYSTEM

We first discuss the absorption spectrum of the unbiased system ($U = 0$), aiming to separate the effects of bias from those related to the disordered nature of the model. Figure 1 represents the absorption spectra calculated for two values of the correlation exponent, $\alpha = 1$ and $\alpha = 4$. By convention, we will refer to these two cases as weakly and strongly long-range correlated disorder, respectively. The results were obtained by numerically diagonalizing the Hamiltonian (1) for chains of size $N = 500$ with open boundary conditions. The disorder magnitude was set to $\sigma = 1$. Averaging over 3×10^4 realizations of the disorder were applied in Eq. (4) for each value of α.

From Fig. 1 we observe that in the case of weak correlations in disorder ($\alpha = 1$), the absorption spectrum consists of a single inhomogeneously broadened asymmetric line. Its red and blue tails can be fitted by a Gaussian and Loretzian, respectively. The spectrum is peaked slightly below the low-energy band edge $E = -2$ and has the FWHM $\sigma^* \approx 2.2$ which is approximately three times larger than that in the limit of uncorrelated disorder ($\alpha = 0$), Eq. (5b). This tendency is in full accordance with that we mentioned in the preceding section.

The absorption line shape, however, changes dramatically when disorder is strongly long-range correlated

($\alpha = 4$). The higher-energy peak in the absorption spectrum starts to build up when the correlation exponent α exceeds the critical value $\alpha_c = 2$ or, in other words, when a phase of the extended states emerges in the center of the band.[66] Its appearance implies that states deep inside the band gain large oscillator strengths. This is in contrast to the case of uncorrelated disorder, where higher states have a vanishingly small oscillator strength.[71,72]

A simple explanation of this anomaly is based on the fact that for sufficiently large values of α, the first term in the series (2) is dominant, while the others are considerably smaller. Consequently, the site potential for a given realization is cosine-like (harmonic with $k = 1$), perturbed by a colored noise (harmonics with $k \geq 2$). Then, the whole lattice can be represented as a two weakly-coupled sublattices with different site energy (see Ref. 66 for more details). Optical transitions to the band edge states of these two sublattices give rise to a double-peaked structure of the absorption spectrum. Remarkably, the higher-energy peak monitors the upper mobility edge of the delocalized phase.[66]

V. BIASED SYSTEM

A. Qualitative picture

In disorder-free systems, switching the bias on results in dynamical (Bloch) localization of all the states[37] within the localization size $L_B = 4/U$, where 4 is the band width in dimensionless units. The Bloch localization is accompanied by the subsequent reorganization of the energy spectrum of the system, which becomes ladder-like with the level spacing U.[56] This structure is revealed in photoluminescence[59,60] and photoconductivity[62] spectra as a series of equally spaced peaks. Disorder broadens the peaks and makes them unresolved. Below we derive a relationship between magnitudes of disorder σ and bias U, which governs the occurrence of the WSL in disordered systems.

Briefly, our reasoning is as follows (a detailed study will be published elsewhere[75]). According to the exchange narrowing concept (see the discussion in Sec. III and Refs. 69,71,72,73,74), a quasiparticle confined within a chain segment of size L_B sees a disorder of a reduced magnitude $\sigma/\sqrt{L_B}$, with σ the strength of the bare disorder. Therefore, the resulting inhomogeneous broadening of the WSL levels can be estimated, similarly to Eq. (5b), as

$$\sigma_B = \frac{2\sigma}{\sqrt{L_B}} = \sigma\sqrt{U} \ . \qquad (6)$$

In order to resolve the ladder structure, the inhomogeneous width σ_B must be smaller than the level spacing U. This brings us to the condition,

$$\sigma < \sqrt{U} \ , \qquad (7)$$

which governs the occurrence of the WSL in 1D disordered systems. We will refer to the relationship (7) as to the limit of strong bias.

Now, we turn to discussing the opposite sign of inequality (7), which we will name the low or/and moderate bias limit. At non zero bias, each site gets an additional energy $-U(n - N/2)$. Thus, the energy difference between the edge sites (magnitude of the total bias) is about UN. It is to be compared first of all to the absorption line width σ^* in the absence of bias. Apparently, at $UN \ll \sigma^*$, i.e., when the potential drop across the whole system is much smaller than the width, the effect of bias on the absorption is negligible. On the contrary, the bias is expected to broaden the absorption spectrum when $UN \gg \sigma^*$. Indeed, optically dominant (bell-like) states in this case will be distributed from $[-UN/2$ to $UN/2]$, giving rise to almost constant absorption within this energy range.

Such a scenario, however, holds as long as $UN^* < \sigma^*$, i.e., provided the typical potential drop across the potential wells supporting bell-like states is smaller than the typical well depth σ^* in bias-free systems. At $UN^* > \sigma^*$, the picture of localization in the potential wells does not work anymore. The localization of states now is governed by a complicated interplay of disorder and bias, which is hard to handle qualitatively. Nevertheless, we can still claim that the FWHM of the absorption spectrum will be on the order of magnitude of the total bias, UN. On further increasing the bias, we fall in the regime of the WSL, $\sqrt{U} > \sigma$. Our numerical simulation confirm this qualitative picture, as shown below.

Note that the above reasonings can be applied without any remarks to both uncorrelated and weakly correlated disorder ($\alpha < 1$). In the limit of strong correlations ($\alpha \gg 1$) the picture requires corrections which we discuss in Sec. V C.

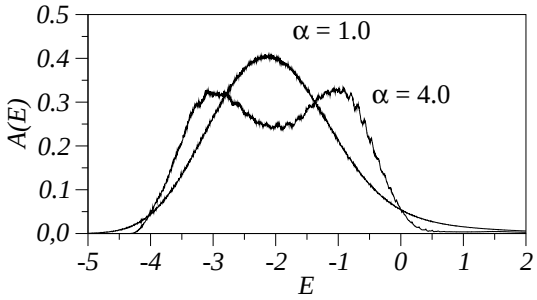

FIG. 2: Absorption spectra of biased chains ($U = 0.01$) with $N = 100$ sites calculated for two values of the correlation exponent α, shown in the plot. The magnitude of disorder is $\sigma = 1$. Each curve were obtained after averaging over 10^6 realizations of disorder.

B. Low and moderate bias

In Fig. 2 we plotted the absorption spectra calculated for disordered biased chains of $N = 100$ sites, choosing the disorder strength $\sigma = 1$ and the bias magnitude $U = 0.01$. Averaging over 10^6 realizations of disorder were performed in Eq. (4). Two values of the correlation exponents were considered, $\alpha = 1$ and $\alpha = 4$. As $\sigma \gg \sqrt{U}$, we are not in the WSL regime. Furthermore, at $U = 0.01$ the overall potential drop $UN = 1$ is smaller than the FWHM in the absence of bias, $\sigma^* \approx 2.2$ (see Fig. 1). As a consequence, the effect of bias is weak, leading only to a unnoticeable broadening of the spectra and smoothing the shape of the doublet (at $\alpha = 4$) as compared to the bias-free conditions (compare to Fig. 1).

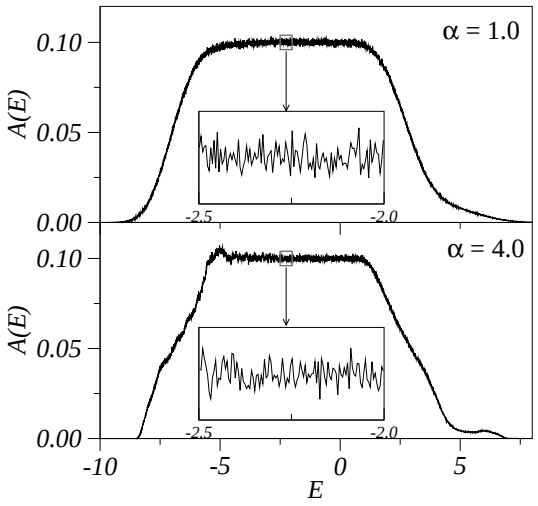

FIG. 3: Absorption spectra of biased chains ($U = 0.1$) with $N = 100$ sites calculated for two values of the correlation exponent α, shown in the plot. The magnitude of disorder is $\sigma = 1$. Each curve were obtained after averaging over 10^6 realizations of disorder. Insets show enlarged views of the spectra within the grey boxes.

Figure 3 presents the results of the simulations for a larger magnitude of the bias, $U = 0.1$, keeping all other parameters unchanged. Still, $\sigma > \sqrt{U}$ that is not in favor of the WSL. However, the total bias $UN = 10$ is now larger than the bias-free FWHM ≈ 2.2. Therefore, for both values of the correlation exponent $\alpha = 1$ and $\alpha = 4$ the absorption spectrum shows a plateau-like shape and a large broadening, with the FWHM ≈ 10 equal to the total bias. These trends are in full agreement with our qualitative reasoning presented in the preceding section.

C. Strong bias

Aiming to find fingerprints of the WSL in the optical absorption spectra, we further increased the magnitude of the bias. Figure 4 shows the spectra calculated for the bias magnitude $U = 0.5$. As before, chains of $N = 100$ sites were used in the simulations and two values of the correlation exponent α were considered, $\alpha = 1$ and $\alpha = 4$. The disorder strength was set to $\sigma = 1$, and averaging over 10^6 realization of disorder was performed in Eq. (4).

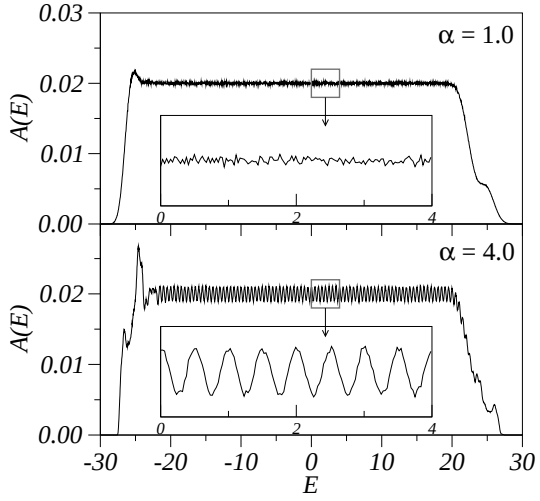

FIG. 4: Same as in Fig. 3, but for a bias $U = 0.5$.

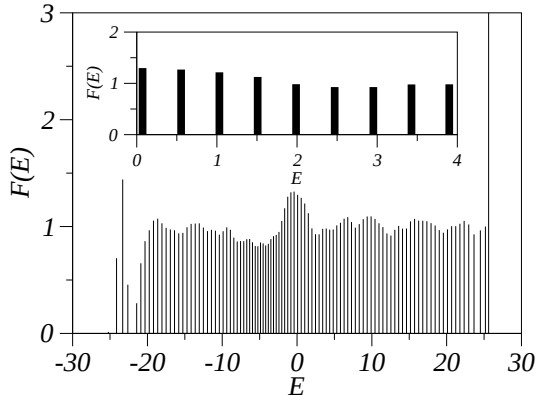

FIG. 5: The oscillator strength distribution for the set of parameters as in lower panel of Fig. 4.

We observe (the upper panel) that at $\alpha = 1 < \alpha_c = 2$ the spectrum remains structureless that is consistent with the estimate (7): still $\sigma > \sqrt{U}$, i.e., the WSL can not be resolved. However, for strong correlations in disorder, when $\alpha = 4 > \alpha_c = 2$, the spectrum presents a periodic pattern which is not masked by the stochastic disorder fluctuations (see the inset in the lower panel). Most important, the period of the modulation is exactly equal to $U = 0.5$. To illustrate more our statement, we depicted in Fig. 5 the oscillator strength F_ν as a function of the eigenenergy E_ν for $\alpha = 4$. It is clearly seen the periodicity in energy with a period equal to $U = 0.5$ as well as the similarity of the corresponding oscillator strengths. We checked out that the pattern with the same spacing also holds for larger systems, i.e, its period is not a consequence of finite size effects. Therefore, we claim that the periodic pattern found in the simulations results from

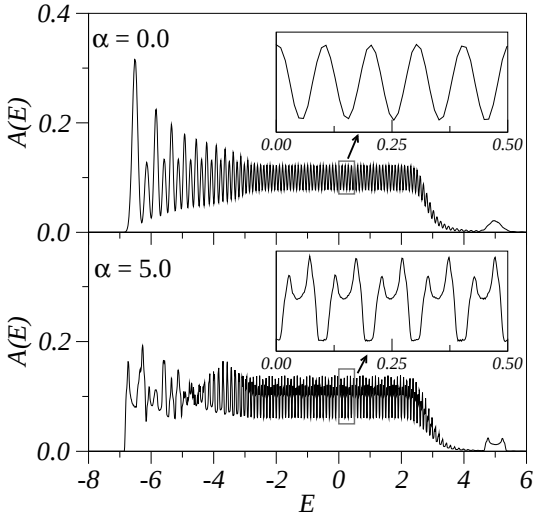

FIG. 6: Same as in Fig. 3, but for a disorder magnitude $\sigma = 0.2$. The inset shows an enlarged view of the center of the band.

the occurrence of the WSL in the energy spectrum of the system.

An explanation of the occurrence of the WSL in the strong correlation regime ($\alpha = 4$) is based on the already mentioned in Sec. IV fact: the site potential (2) for a given realization of disorder is cosine-like (harmonic with $k = 1$), perturbed by a colored noise (harmonics with $k \geq 2$). This implies that the magnitude of the actual disorder is at least by a factor of $2^{-\alpha/2}$ smaller than the bare value σ; the inequality (7) turns out to be fulfilled for the reduced disorder. In other words, strong correlations in disorder facilitate the occurrence of the WSL.

Finally, we calculated the absorption spectrum profile for magnitudes of disorder $\sigma = 0.2$ and bias $U = 0.1$, when the inequality (7) holds even in the absence of correlations in disorder ($\alpha = 0$). The results, obtained for $\alpha = 0$ and $\alpha = 5$ are depicted in Fig. 6 (upper and lower panels, respectively). One observes that the absorption spectrum shows a resolved structure for both values of α, which, however, is heterogeneous. The central part of the spectrum is of our primary interest because it reveals a periodic pattern.

At $\alpha = 0$ (uncorrelated disorder), the pattern consists of equally spaced single peaks with the spacing exactly equal to $U = 0.1$ (see the inset). This again allows us to associate the peaks with the occurrence of WSL in the energy spectrum of the system. The FWHM of a single peak is about 0.06 that is in good agreement with Eq. (6). We checked also out that the pattern did not appear if $\sigma = 0.4 \approx \sqrt{U} = 0.45$, in full accordance with our reasonings presented in Sec. V A. These results corroborate those found in Ref. 65 for random Kronig-Penney models.

In the case of strongly correlated disorder ($\alpha = 5$), each single peak of the central pattern splits into a doublet, as is seen from the inset in the lower panel. The origin of the doublets can be traced back to the behavior of the unbiased system, where the absorption line shape exhibits a doublet structure provided $\alpha > \alpha_c = 2$ (see Sec. IV and Ref. 66). Figure 6 points out that the splitting also occurs in the presence of bias.

To conclude we comment on the peculiarities of the red and blue sides of the absorption spectrum. We associate them with finite size efects. Indeed, these parts of the energy spectrum are formed by the states localized close to the system ends. Because of that, the corresponding eigenfunctions differs from those at the band center.

VI. SUMMARY AND CONCLUDING REMARKS

We studied numerically the linear optical absorption of a quasiparticle moving on a 1D disordered lattice subjected to a linear bias of magnitude U. The random site potential was set to have a power-law spectral density $S(k) \sim 1/k^{\alpha}$, which gives rise to long-range correlations in site energies.

The absorption spectrum of the unbiased lattice ($U = 0$) was found to present a single peak in the weakly-correlated limit, when the correlation exponent $\alpha < \alpha_c = 2$, with $\alpha_c = 2$ the critical value for the occurrence of a phase of extended states in the center of the band. For strongly-correlated disorder, at $\alpha > \alpha_c = 2$, the absorption lineshape turned out to exhibit a double-peaked structure, characteristic for this type of long-range correlations.[66]

Switching the bias on does not change much the outlined behavior provided the magnitude of overall bias UN does not exceed the absorption bandwidth of the unbiased system. At higher magnitudes of bias, the absorption spectrum starts to broaden, independently of the magnitude of the correlation exponent α. Its profile gets a top hat shape, being almost flat at the center of the absorption band and having the FWHM on the order of UN. Such a scenario holds as long as the disorder magnitude $\sigma < \sqrt{U}$.

On further increasing the magnitude of the bias, a periodic pattern is found to build up at the center of the (already wide) absorption band. Its period is equal to U, as for the Wanier-Stark ladder in an ideal lattice, and independent of the system size N. Therefore, we attribute the pattern found to the Wannier-Stark quantization of the energy spectrum in the disordered lattice.

The occurrence of the Wannier-Stark pattern is facilitated by the presence of correlations in disorder. In the limit of strong correlations ($\alpha > \alpha_c = 2$), each Wannier-Stark level represents a doublet, reflecting the doublet structure of the absorption spectrum of the unbiased system.

To conclude we note that in our study we did not explicitly relate the bias to the presence of an external uniform electric field. Therefore, our conclusions can be equally applied both to 1D disordered electrons, moving

in a uniform electric field, and to 1D disordered Frenkel excitons, where energetic bias can be an intrinsic property of the system. Dendrimers represent one of the examples.[68]

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